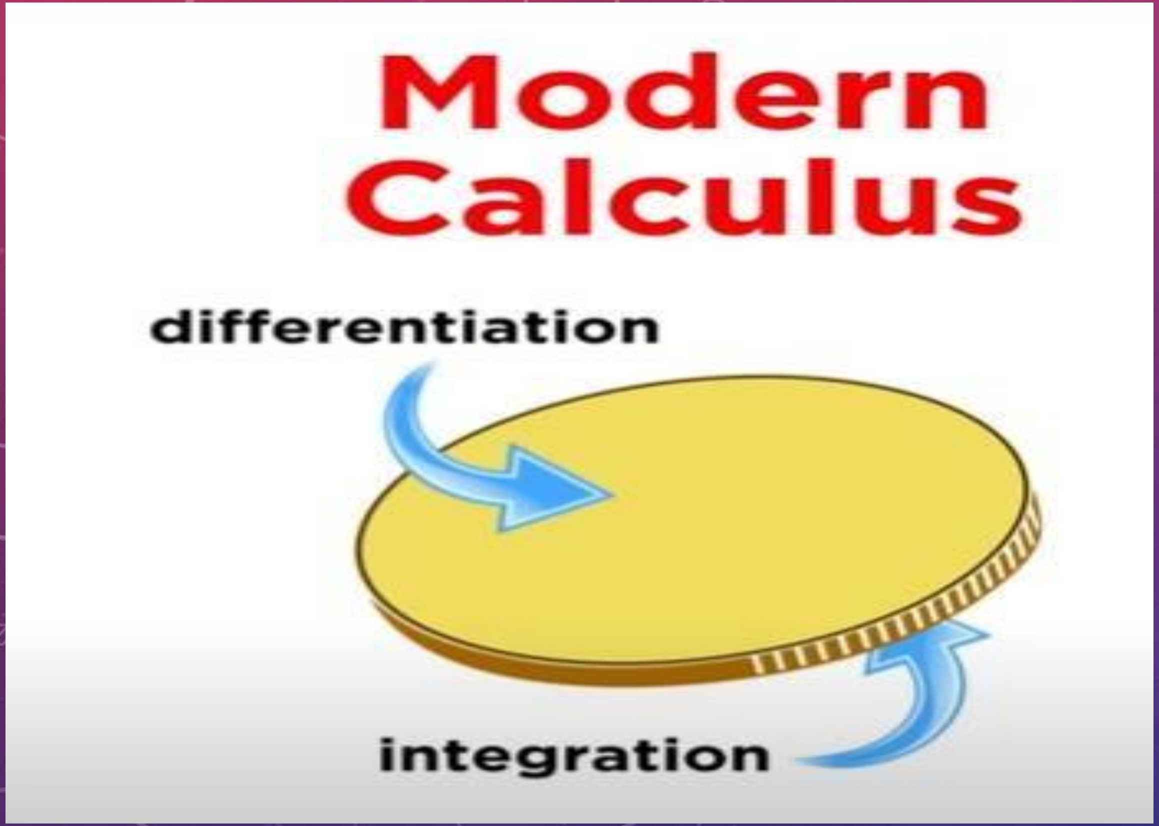
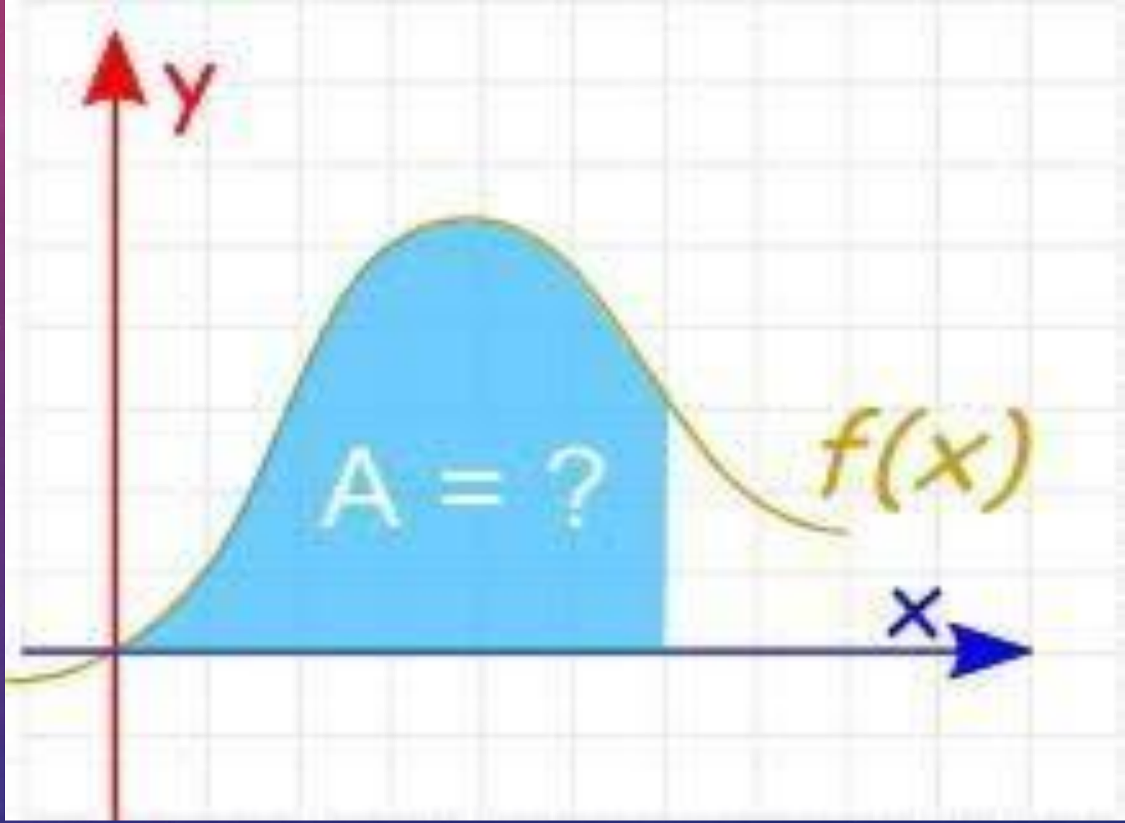
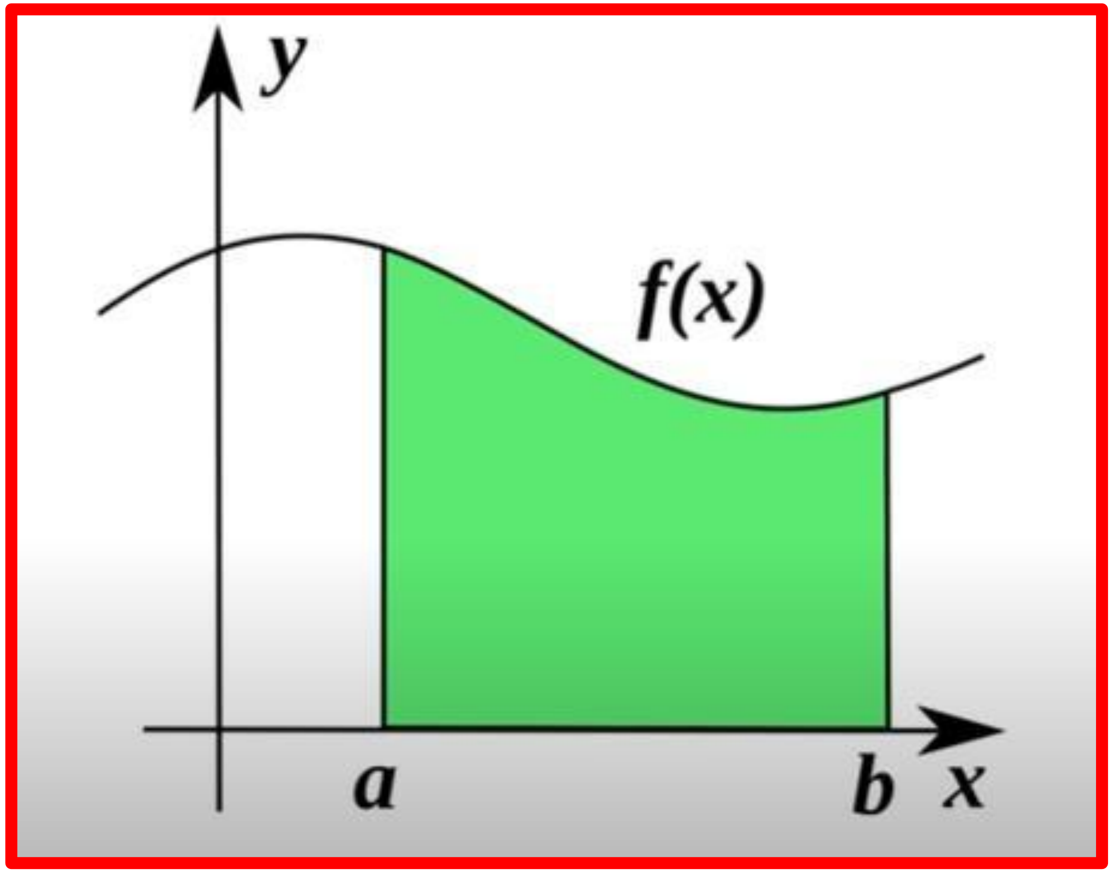
INTEGRAL CALCULUS







$$A = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

$f(x_i)$ is the height of each rectangle

Δx is the width of each rectangle

Adding up an infinite number of rectangles gives us the area under the curve

Integration as an Inverse Process of Differentiation…….

Example: What is an integral of 2x?

$$\int 2x \, dx = x^2 + C$$

We know that the derivative of x^2 is $2x$ …

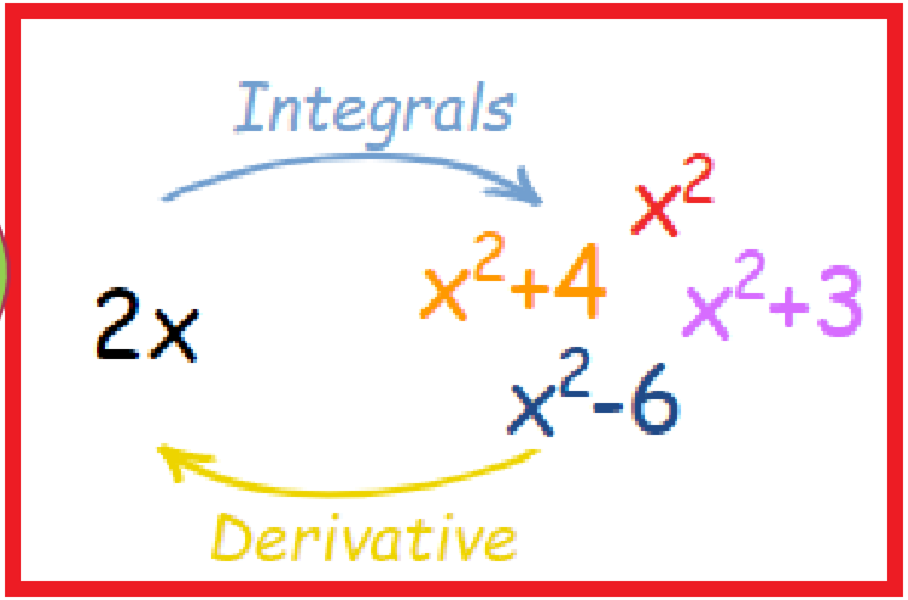
… so an integral of $2x$ is x^2

We wrote the answer as x^2 but why $+C$?

$\frac{d}{dx}(x^2+4) = 2x$, $\frac{d}{dx}(x^2-6) = 2x$ so on…

When we **reverse** the operation (to find integral), we only know $\mathbf{2x}$, but there could be a constant of any value…..

So we just write $+C$ at the end .



PROPERTIES OF INDEFINITE INTEGRAL

Rules	Function	Integral
Multiplication by constant	$\int cf(x)\,dx$	$c\int f(x)\,dx$
Power Rule (n≠-1)	$\int x^n\,dx$	$\frac{x^{n+1}}{n+1} + C$
Sum Rule	$\int (f + g)\,dx$	$\int f\,dx + \int g\,dx$
Difference Rule	$\int (f - g)\,dx$	$\int f\,dx - \int g\,dx$

ANTI-DERIVATIVES

INTEGRATION FORMULAS

DIFFERENTIATION FORMULA	INTEGRATION FORMULA	DIFFERENTIATION FORMULA	INTEGRATION FORMULA
1. $\frac{d}{dx}[x] = 1$	$\int dx = x + C$ ($\int k\, dx = kx + C$)	8. $\frac{d}{dx}[-\csc x] = \csc x \cot x$	$\int \csc x \cot x\, dx = -\csc x + C$
2. $\frac{d}{dx}\left[\frac{x^{r+1}}{r+1}\right] = x^r \quad (r \neq -1)$	$\int x^r\, dx = \frac{x^{r+1}}{r+1} + C \quad (r \neq -1)$	9. $\frac{d}{dx}[e^x] = e^x$	$\int e^x\, dx = e^x + C$
3. $\frac{d}{dx}[\sin x] = \cos x$	$\int \cos x\, dx = \sin x + C$	10. $\frac{d}{dx}\left[\frac{b^x}{\ln b}\right] = b^x \quad (0 < b, b \neq 1)$	$\int b^x\, dx = \frac{b^x}{\ln b} + C \quad (0 < b, b \neq 1)$
4. $\frac{d}{dx}[-\cos x] = \sin x$	$\int \sin x\, dx = -\cos x + C$	11. $\frac{d}{dx}[\ln \lvert x \rvert] = \frac{1}{x}$	$\int \frac{1}{x}\, dx = \ln \lvert x \rvert + C$
5. $\frac{d}{dx}[\tan x] = \sec^2 x$	$\int \sec^2 x\, dx = \tan x + C$	12. $\frac{d}{dx}[\tan^{-1} x] = \frac{1}{1+x^2}$	$\int \frac{1}{1+x^2}\, dx = \tan^{-1} x + C$
6. $\frac{d}{dx}[-\cot x] = \csc^2 x$	$\int \csc^2 x\, dx = -\cot x + C$	13. $\frac{d}{dx}[\sin^{-1} x] = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{1-x^2}}\, dx = \sin^{-1} x + C$
7. $\frac{d}{dx}[\sec x] = \sec x \tan x$	$\int \sec x \tan x\, dx = \sec x + C$	14. $\frac{d}{dx}[\sec^{-1} \lvert x \rvert] = \frac{1}{x\sqrt{x^2-1}}$	$\int \frac{1}{x\sqrt{x^2-1}}\, dx = \sec^{-1} \lvert x \rvert + C$

Find the anti derivative(or integral) of the following functions:

Ex 7.1, 1

Find anti derivative of sin 2x

We know that

$$(\cos 2x)' = \sin 2x \,.\, (-2)$$

$$\frac{-1}{2}(\cos\ 2x)' = \sin 2x$$

$$\sin 2x = \left(\frac{-1}{2}\cos 2x\right)'$$

$\therefore$ Anti derivate of $\sin 2x = \frac{-1}{2}\ \mathbf{\cos 2x}$

Ex 7.1, 4

$(ax+b)^2$

We know that

$$\left((ax+b)^3\right)' = 3(ax+b)^{3-1} \,.\, \frac{d(ax+b)}{dx}$$

$$\left((ax+b)^3\right)' = 3(ax+b)^2(a.1+0)$$

$$\left((ax+b)^3\right)' = 3a(ax+b)^2$$

$$\frac{1}{3a}\left((ax+b)^3\right)' = (ax+b)^2$$

$$(ax+b)^2 = \left(\frac{1}{3a}(ax+b)^3\right)'$$

$\Rightarrow$ Anti derivate of $(ax+b)^2 = \frac{1}{3a}\mathbf{(ax+b)^3}$

Ex 7.1, 6

$\int (4e^{3x} + 1)\, dx$

$$\int (4e^{3x} + 1)\, dx$$

$$= \int (4e^{3x} + x^0)\, dx$$

$$= 4\int e^{3x}\, dx + \int x^0\, dx$$

$$= \frac{4e^{3x}}{3} + \frac{x^{0+1}}{0+1} + C$$

$$= \frac{4e^{3x}}{3} + x + C$$

As
$\int e^x\, dx = e^x + C$
$\int x^n\, dx = \frac{x^{n+1}}{n+1} + C$

Ex 7.1, 10

$\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 dx$

$$\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 dx$$

$$= \int \left((\sqrt{x})^2 + \left(\frac{1}{\sqrt{x}}\right)^2 - 2(\sqrt{x})\left(\frac{1}{\sqrt{x}}\right)\right) dx$$

$$= \int \left(x + \frac{1}{x} - 2\right) dx$$

$$= \int \left(x + \frac{1}{x} - 2x^0\right) dx$$

$$= \int x\, dx + \int \frac{1}{x}\, dx - 2\int x^0\, dx$$

$$= \frac{x^{1+1}}{1+1} + \log|x| - \frac{2x^{0+1}}{0+1} + C$$

$$= \frac{x^2}{2} + \log|x| - 2x + C$$

As
$\int x^n\, dx = \frac{x^{n+1}}{n+1} + C$
$\int \frac{1}{x}\, dx = \log|x| + C$

Ex 7.1, 13

$$\int \frac{x^3 - x^2 + x - 1}{x - 1} dx$$

$$\int \frac{x^3 - x^2 + x - 1}{x - 1} dx$$

$$= \int \frac{x^2 (x - 1) + 1 (x - 1)}{x - 1} dx$$

$$= \int \frac{(x^2 + 1)(x - 1)}{x - 1} dx$$

$$= \int (x^2 + 1)\, dx$$

$$= \int (x^2 + x^0)\, dx$$

$$= \int x^2\, dx + \int x^0\, dx$$

$$= \frac{x^{2+1}}{2+1} + \frac{x^{0+1}}{0+1} + C \qquad \left(\int x^n\, dx = \frac{x^{n+1}}{n+1}\right)$$

$$= \boldsymbol{\frac{x^3}{3} + x + C}$$

Ex 7.1, 18

Find anti derivative of $\int \sec x\,(\sec x + \tan x)dx$

$$\int \sec x\,(\sec x + \tan x)\, dx$$

$$= \int (\sec^2 x + \sec x \tan x)\, dx$$

$$= \int \sec^2 x\, dx + \int (\sec x \tan x)\, dx$$

$$= \boldsymbol{\tan x + \sec x + C}$$

As

$\int \sec^2 x\, dx = \tan x + C$

& $\int \sec x \tan x\, dx = \sec x + C$

Ex 7.1, 22

If $\frac{d}{dx}$ f(x) = $4x^3 - \frac{3}{x^4}$ such that f(2) = 0, then f(x) is

(A) $x^4 + \frac{1}{x^3} - \frac{129}{8}$

(B) $x^3 + \frac{1}{x^4} + \frac{129}{8}$

(C) $x^4 + \frac{1}{x^3} + \frac{129}{8}$

(D) $x^3 + \frac{1}{x^4} - \frac{129}{8}$

Given

$$\frac{d}{dx} f(x) = 4x^3 - \frac{3}{x^4}$$

Integrating both sides

$$\int \frac{d}{dx} f(x) = \int \left(4x^3 - \frac{3}{x^4}\right) dx$$

$$\int \frac{d}{dx} f(x) = 4\int x^3\, dx - 3\int \frac{1}{x^4}\, dx$$

$$f(x) = 4\int x^3 dx - 3\int x^{-4}\, dx$$

$$f(x) = 4\frac{x^{3+1}}{3+1} - 3\frac{x^{-4+1}}{-4+1} + C \qquad \left(\text{As} \int x^n\, dx = \frac{x^{n+1}}{n+1} + C\right)$$

$$f(x) = 4\frac{x^4}{4} - 3\frac{x^{-3}}{-3} + C$$

$$f(x) = x^4 + \frac{1}{x^3} + C \qquad ...(1)$$

Given $f(2) = 0$

Putting $x = 2$ in (1)

$$f(2) = (2)^4 + \frac{1}{(2)^3} + C$$

$$0 = \frac{129}{8} + C \qquad C = \frac{-129}{8}$$

Putting $C = \frac{-129}{8}$ in (1)

$$f(x) = x^4 + \frac{1}{x^3} + C$$

$$\Rightarrow \boldsymbol{f(x) = x^4 + \frac{1}{x^3} - \frac{129}{8}}$$

∴ **Option (A)** is correct.

ANTI - DERIVATIVES

THE REAL FEEL

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"Two roads diverged in woods and
I took the one less travelled by,
and that has made all the difference"
ROBERT FROST

INTEGRATION FORMULAS

DIFFERENTIATION FORMULA	INTEGRATION FORMULA
1. $\frac{d}{dx}[x] = 1$	$\int dx = x + C$
2. $\frac{d}{dx}\left[\frac{x^{r+1}}{r+1}\right] = x^r \quad (r \neq -1)$	$\int x^r\,dx = \frac{x^{r+1}}{r+1} + C \quad (r \neq -1)$
3. $\frac{d}{dx}[\sin x] = \cos x$	$\int \cos x\,dx = \sin x + C$
4. $\frac{d}{dx}[-\cos x] = \sin x$	$\int \sin x\,dx = -\cos x + C$
5. $\frac{d}{dx}[\tan x] = \sec^2 x$	$\int \sec^2 x\,dx = \tan x + C$
6. $\frac{d}{dx}[-\cot x] = \csc^2 x$	$\int \csc^2 x\,dx = -\cot x + C$
7. $\frac{d}{dx}[\sec x] = \sec x \tan x$	$\int \sec x \tan x\,dx = \sec x + C$

DIFFERENTIATION FORMULA	INTEGRATION FORMULA
8. $\frac{d}{dx}[-\csc x] = \csc x \cot x$	$\int \csc x \cot x\,dx = -\csc x + C$
9. $\frac{d}{dx}[e^x] = e^x$	$\int e^x\,dx = e^x + C$
10. $\frac{d}{dx}\left[\frac{b^x}{\ln b}\right] = b^x \quad (0 < b, b \neq 1)$	$\int b^x\,dx = \frac{b^x}{\ln b} + C \quad (0 < b, b \neq 1)$
11. $\frac{d}{dx}[\ln\|x\|] = \frac{1}{x}$	$\int \frac{1}{x}\,dx = \ln\|x\| + C$
12. $\frac{d}{dx}[\tan^{-1} x] = \frac{1}{1+x^2}$	$\int \frac{1}{1+x^2}\,dx = \tan^{-1} x + C$
13. $\frac{d}{dx}[\sin^{-1} x] = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C$
14. $\frac{d}{dx}[\sec^{-1}\|x\|] = \frac{1}{x\sqrt{x^2-1}}$	$\int \frac{1}{x\sqrt{x^2-1}}\,dx = \sec^{-1}\|x\| + C$

Suppose that $F(x)$ and $G(x)$ are antiderivatives of $f(x)$ and $g(x)$, respectively, and that c is a constant. Then:

(a) *A constant factor can be moved through an integral sign; that is,*

$$\int cf(x)\,dx = cF(x) + C$$

(b) *An antiderivative of a sum is the sum of the antiderivatives; that is,*

$$\int [f(x) + g(x)]\,dx = F(x) + G(x) + C$$

(c) *An antiderivative of a difference is the difference of the antiderivatives; that is,*

$$\int [f(x) - g(x)]\,dx = F(x) - G(x) + C$$

THE INDEFINITE INTEGRAL

The process of finding antiderivatives is called ***antidifferentiation*** or ***integration***. Thus, if

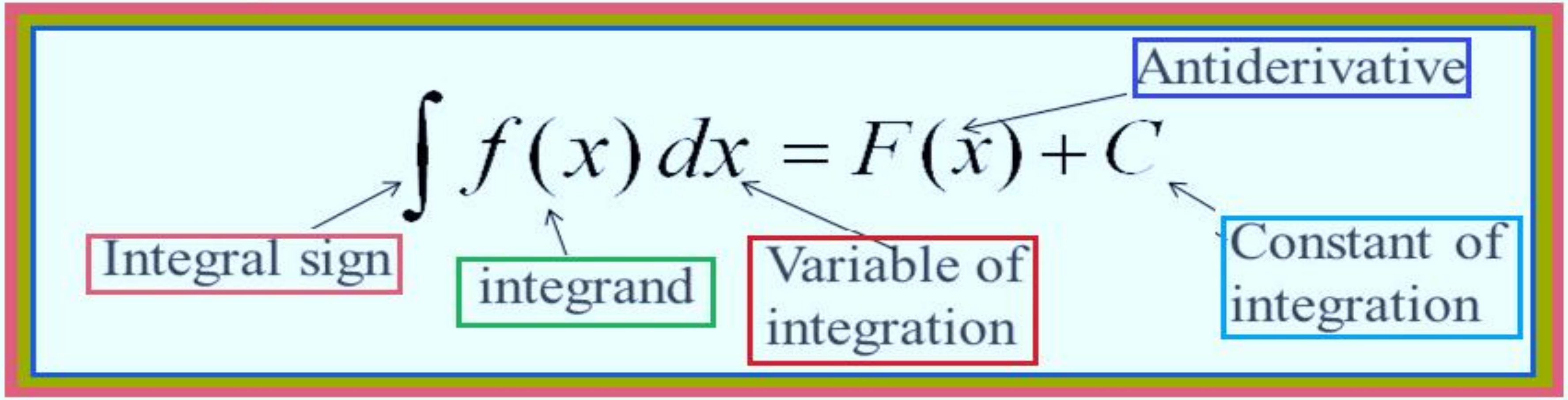
$$\frac{d}{dx}[F(x)] = f(x) \qquad (1)$$

then ***integrating*** (or ***antidifferentiating***) the function $f(x)$ produces an antiderivative of the form $F(x) + C$. To emphasize this process, Equation (1) is recast using ***integral notation***,

$$\int f(x)\,dx = F(x) + C \qquad (2)$$

where C is understood to represent an arbitrary constant. It is important to note that (1) and (2) are just different notations to express the same fact. For example,

$$\int x^2\,dx = \tfrac{1}{3}x^3 + C \quad \text{is equivalent to} \quad \frac{d}{dx}\left[\tfrac{1}{3}x^3\right] = x^2$$



Equation (2) should be read as:

The integral of $f(x)$ with respect to x is equal to $F(x)$ plus a constant.

Find $\int x^3 \, dx$

Solution:

Step 1: Try $f(x) = x^4$ (now this is to be thought of)

Step 2: $f'(x) = 4x^3$ (now 4 is not part of the question so how to remove it)

Step 3: Not quite right. Correct power, wrong constant.

Step 4: Try $f(x) = \frac{1}{4}x^4$ $f'(x) = x^3$ Correct!

Thus $\int x^3 \, dx = \frac{1}{4}x^4$

Evaluate $\int (3x^2+4x+5)\,dx$.

Solution

What functions have a derivative of $3x^2$? Some thought will lead us to a cubic, specifically x^3+C_1, where C_1 is a constant.

What functions have a derivative of $4x$?

Here the x term is raised to the first power, so we likely seek a quadratic. $2x^2+C_2$, where C_2 is a constant.

Finally, what functions have a derivative of 5?

Functions of the form $5x+C_3$, where C_3 is a constant.

Our answer appears to be

$$\int (3x^2+4x+5)\,dx = x^3+C_1+2x^2+C_2+5x+C_3.$$

We do not need three separate constants of integration; combine them as one constant, giving the final answer of

$$\int (3x^2+4x+5)\,dx = x^3+2x^2+5x+C.$$

▶ Evaluate $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx.$

Consider: $e^{\sqrt{x}}$

lets differentiate $e^{\sqrt{x}}$ **(why ?)**

$$\frac{d}{dx} e^{\sqrt{x}} = e^{\sqrt{x}} \frac{1}{2\sqrt{x}}$$

$$2 \cdot \frac{d}{dx} e^{\sqrt{x}} = e^{\sqrt{x}} \frac{1}{\sqrt{x}}$$

$$2 \cdot d\, e^{\sqrt{x}} = \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx.$$

$$d\,(2 \cdot e^{\sqrt{x}}) = \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx.$$

$$d\,(2 \cdot e^{\sqrt{x}}) + 0 = \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx$$

$$\int d\,(2 \cdot e^{\sqrt{x}} + c) = \int \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx$$

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx = 2 \cdot e^{\sqrt{x}} + c$$

► **Example** Evaluate $\int t^4 \sqrt[3]{3-5t^5}\,dt.$

Consider $(3-5t^5)^{4/3}$ (why only this function)

lets differentiate $(3-5t^5)^{4/3}$

$$\frac{d(3-5t^5)^{4/3}}{dx} = \frac{4}{3}(3-5t^5)^{1/3}(0-25t^4)$$

$$= (-25)\frac{4}{3}t^4\,(3-5t^5)^{1/3}$$

$$d(3-5t^5)^{4/3} + 0 = -\frac{3}{100}\, t^4\sqrt[3]{3-5t^5}\,dx$$

$$-\frac{100}{3}\int d(3-5t^5)^{4/3} + c = \int t^4\sqrt[3]{3-5t^5}\,dx$$

$$\int t^4\sqrt[3]{3-5t^5}\,dx = -\frac{100}{3}\int d(3-5t^5)^{4/3} + c$$

$$\int t^4\sqrt[3]{3-5t^5}\,dx = -\frac{100}{3}(3-5t^5)^{4/3} + c$$

Evaluate $\int x \cos x \, dx$.

Now differentiation of sinx is cos x . Since x is multiplied with cos x , let us differentiate (x sinx)

d(x sinx) = x d(sinx) + sin x d(x)
d(x sinx) = x cosx + sinx . 1

x cosx = d(x sinx) – sinx

When you integrate both sides wrt ' x' we get

$$\int x \cos x \, dx = x \sin x + \cos x + C,$$

1. (a) $\int (5-2x)^8\,dx$

(b) $\int \frac{\sin x}{\sqrt{2+\cos x}}\,dx$

(c) $\int \tan^2 x \sec^2 x\,dx$

2. (a) $\int \frac{\sqrt{\tan^{-1} x}}{1+x^2}\,dx$

(b) $\int \frac{\ln x}{x}\,dx$

Intuition is more powerful than intellect

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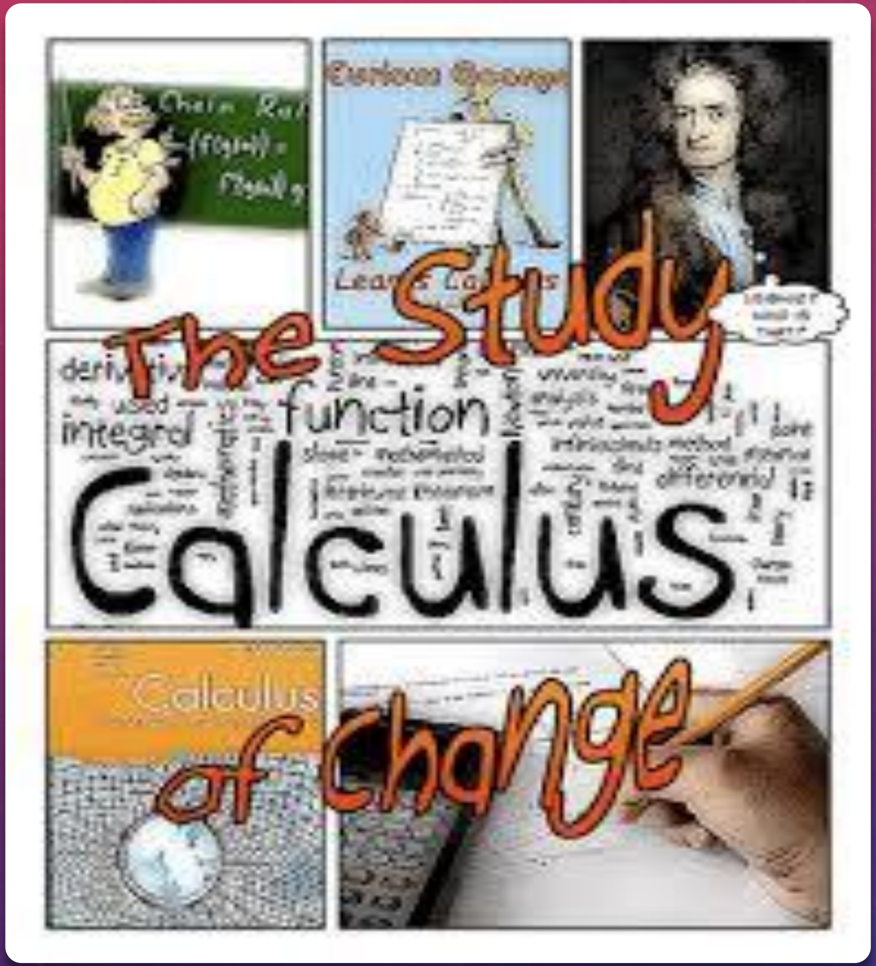

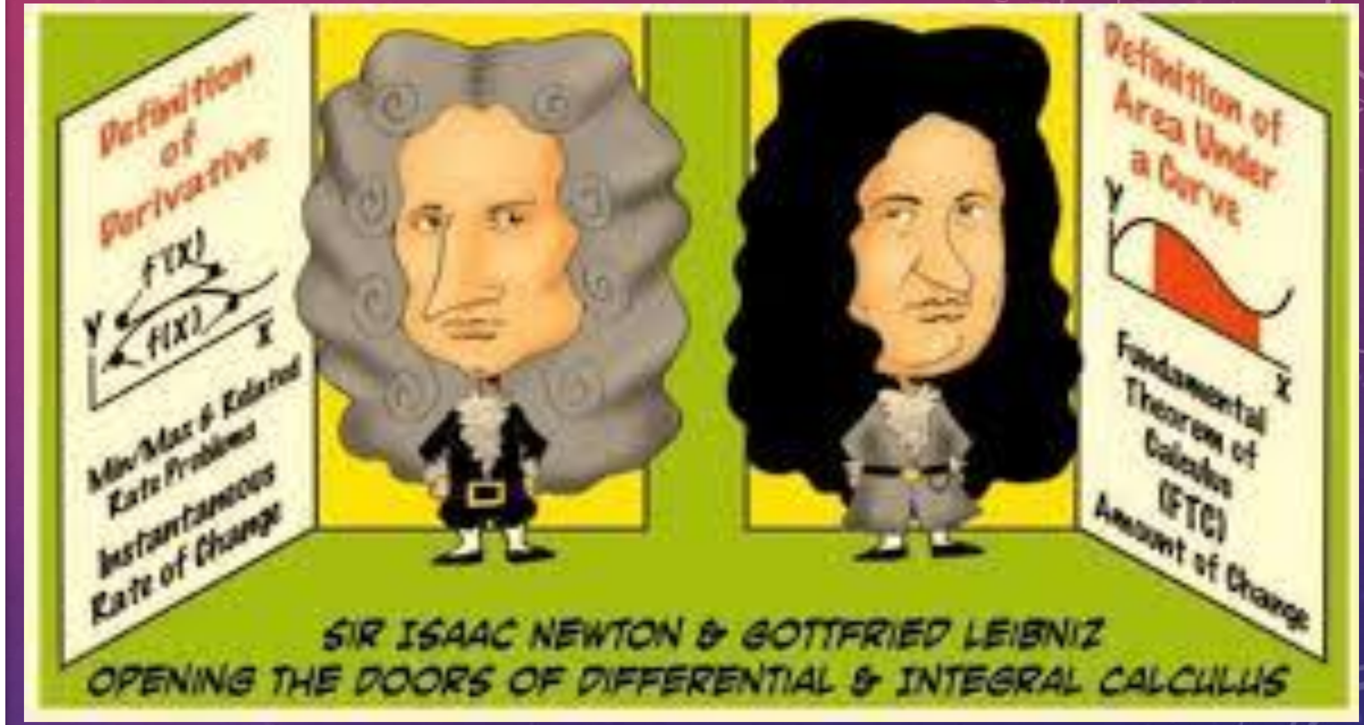

integrals

Module - 3

Here's What Integration is!

If $\frac{d}{dx}(F(x)) = f(x)$, then

$$\int f(x)\,dx = F(x) + c$$

The function F(x) is called anti-derivative or integral or primitive of the given function f(x) and c is known as the constant of integration or the arbitrary constant.

The function f(x) is called the integrand and f(x)dx is known as the element of integration.

Points to Remember:

Since the integral of a function isn't definite, therefore it is generally referred to as indefinite integral.
We can never find the integral of a function at a point; we always find the integral of a given function in an interval.
Integral of a function is not unique; integrals of a function differ by numbers.

BASIC INTEGRATION FORMULAE ......

1. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, n \neq -1.$ $Particularly, \int dx = x + c$
2. $\int \cos x\ dx = \sin x + C$
3. $\int \sin x\ dx = -\cos x + C$
4. $\int \sec^2 x\,dx = \tan x + c$
5. $\int cosec^2 x\,dx = -\cot x + c$
6. $\int \sec x \tan x\,dx = \sec x + c$
7. $\int cosec\,x \cot x\,dx = -cosec\,x + c$
8. $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + c$
9. $\int \frac{dx}{\sqrt{1-x^2}} = -\cos^{-1} x + c$
10. $\int \frac{dx}{1+x^2} = \tan^{-1} x + c$
11. $\int \frac{dx}{1+x^2} = -\cot^{-1} x + c$
12. $\int e^x\,dx = e^x + c$
13. $\int a^x dx = \frac{a^x}{\log a} + c$
14. $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + c$
15. $\int \frac{dx}{x\sqrt{x^2-1}} = -cosec^{-1} x + c$
16. $\int \frac{1}{x} dx = \log|x| + c$
17. $\int \tan x\,dx = \log|\sec x| + c$
18. $\int \cot x\,dx = \log|\sin x| + c$
19. $\int \sec x\,dx = \log|\sec x + \tan x| + c$
20. $\int cosec\,x\,dx = \log|cosec\,x - \cot x| + c$

NOTE :

$$\int f(x)dx = F(x) + c$$

$$\int f(ax+b)dx = \frac{F(ax+b)}{a} + c$$

1. $\int (ax+b)^n\,dx = \frac{1}{a}\cdot\frac{(ax+b)^{n+1}}{n+1} + C, n \neq -1$

2. $\int \frac{1}{ax+b}\,dx = \frac{1}{a}\log|ax+b| + C$

3. $\int e^{ax+b}dx = \frac{1}{a}e^{ax+b} + C$

4. $\int a^{bx+c}\,dx = \frac{1}{b}\cdot\frac{a^{bx+c}}{\log a} + C, a > 0$ and $a \neq 1$

5. $\int \sin(ax+b)\,dx = -\frac{1}{a}\cos(ax+b) + C$

Now….try this….
Integrate $\sin mx$
with respect to x.

1) $\int sec(5-8x)\,tan(5-8x)\,dx = \frac{sec(5-8x)}{-8} + c$

2) $\int cosec^2(7-8x)dx = \frac{\cot(7-8x)}{8} + c$

3) $\int \frac{1}{(2x-6)^{\frac{3}{7}}}\,dx = \frac{(2x-6)^{\frac{4}{7}}}{2.(\frac{4}{7})} + c = \frac{7}{8}(2x-6)^{\frac{4}{7}} + c$

4) $\int \frac{1}{2x+5}\,dx = \frac{1}{2}\log|2x+5| + c$

5) $\int \frac{e^{(9x+2)}}{e^{(3x-2)}}\,dx = \int e^{6x+4}dx = \frac{e^{6x+4}}{6} + c$

6) $\int \frac{5^{7x-5)}}{5^{(2x+10)}}\,dx = \int 5^{5x-15)}dx = \frac{5^{5x-15)}}{5.\log 5} + c$

Here's a list of Integration Methods –

1.Integration by Substitution

2. Integration Using Trigonometric Identities

3.Integration of Some particular fraction

4.Integration by Partial Fraction

5.Integration by Parts

The Substitution Method

According to the substitution method, a given integral ∫ f(x) dx can be transformed into another form by changing the independent variable x to t. This is done by substituting x = g (t).

Consider, I = ∫ f(x) dx
Now, substitute x = g(t) so that, dx/dt = g'(t) or dx = g'(t)dt.
Therefore, I = ∫ f(x) dx = ∫ f[g(t)] g'(t)dt

It is important to note here that you should make the substitution for a function whose derivative also occurs in the integrand as shown in the following examples.

Example 1

Integrate $2x \sin(x^2 + 1)$ with respect to x.

Solution: We know that the derivative of $(x^2 + 1) = 2x$. Hence, let's substitute $(x^2 + 1) = t$, so that $2x = \frac{dt}{dx}$. Therefore, $2x\,dx = dt$

Now,

$$\int 2x \sin(x^2 + 1)\,dx = \int \sin t\,dt$$
$$= -\cos t + C \quad = -\cos(x^2 + 1) + C$$

INTEGRATION BY SUBSTITUTION

Ex 7.2, 1

Integrate the function: $\frac{2x}{1+x^2}$

Let $1 + x^2 = t$

Differentiate $w.r.t.x$

$$2x = \frac{dt}{dx}$$

$$dx = \frac{dt}{2x}$$

Thus, our equation becomes

$$\int \frac{2x}{1+x^2}\, dx = \int \frac{2x}{t} \cdot \frac{dt}{2x}$$

$$= \int \frac{dt}{t}$$

$$= \log|t| + C \qquad \left(\int \frac{1}{x}\, dx = \log|x| + C\right)$$

Putting $t = 1 + x^2$

$$= \log|1 + x^2| + C$$

$$= \log(1 + x^2) + C \qquad \textit{(Since } 1 + x^2 \textit{ is always positive)}$$

Ex 7.2, 3

Integrate the function: $\frac{1}{x + x\log x}$

$$\frac{1}{x + x\log x} = \frac{1}{x(1 + \log x)}$$

Step 1:

Let $1 + \log x = t$

Differentiating both sides $w.r.t.x$

$$0 + \frac{1}{x} = \frac{dt}{dx}$$

$$\frac{1}{x} = \frac{dt}{dx}$$

$$dx = x\ dt$$

Step 2:

Integrating function

$$\int \frac{1}{x + x\log x}\ .dx$$

$$= \int \frac{1}{x\ (1 + \log x)}\ .dx$$

Putting $1 + \log x$ & $dx = x\ dt$

$$= \int \frac{1}{x(t)}\ dt.x$$

$$= \int \frac{1}{t}\ dt \qquad \left(Using \int \frac{1}{x}\ dx = \log|x| + C\right)$$

$$= log|t| + C$$

Putting back $t = 1 + log\ x$

$$= \boldsymbol{log|1 + \log x| + C}$$

Ex 7.2, 7

Integrate the function: $x\sqrt{x+2}$

Let $(x+2) = t$

Differentiating both sides $w.r.t.x$

$$1 + 0 = \frac{dt}{dx}$$

$$1 = \frac{dt}{dx}$$

$$dx = dt$$

Now,

$$\int x\sqrt{x+2}\ .dx$$

$$= \int (t-2)\sqrt{t}\ .dt \quad (Using\ x + 2 = t,\ x = t - 2)$$

$$= \int (t-2)t^{\frac{1}{2}}\ .dt$$

$$= \int \left(t.t^{\frac{1}{2}} - 2.t^{\frac{1}{2}}\right)\ .dt$$

$$= \int \left(t^{\frac{3}{2}} - 2.t^{\frac{1}{2}}\right)\ .dt$$

$$= \frac{t^{\frac{3}{2}+1}}{\frac{3}{2}+1} - 2 . \frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C \qquad \left(Using \int x^n .dx = \frac{x^{n+1}}{n+1}\right)$$

$$= \frac{t^{\frac{5}{2}}}{\frac{5}{2}} - 2 . \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$= \frac{2}{5} t^{\frac{5}{2}} - 2 \times \frac{2}{3} t^{\frac{3}{2}} + C$$

$$= \frac{2}{5} t^{\frac{5}{2}} - \frac{4}{3} t^{\frac{3}{2}} + C$$

Putting back $t = x + 2$

$$= \frac{2}{5} (x+2)^{\frac{5}{2}} - \frac{4}{3} (x+2)^{\frac{3}{2}} + C$$

Ex 7.2, 19

Integrate the function $\frac{e^{2x}-1}{e^{2x}+1}$

Dividing numerator and denominator by e^x, we obtain

$$= \frac{\frac{e^{2x}}{e^x} - \frac{1}{e^x}}{\frac{e^{2x}}{e^x} + \frac{1}{e^x}}$$

$$= \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

Let $e^x + e^{-x} = t$

Differentiating both sides $w.r.t. x$

$$e^x + (-1)e^{-x} = \frac{dt}{dx}$$

$$e^x - e^{-x} = \frac{dt}{dx}$$

$$dx = \frac{dt}{e^x - e^{-x}}$$

Now,

$$\int \frac{e^{2x}-1}{e^{2x}+1} \cdot dx$$

$$= \int \frac{e^x - e^{-x}}{e^x + e^{-x}} \cdot dx$$

Putting $e^x + e^{-x} = t$ & $dx = \frac{dt}{e^x - e^{-x}}$

$$= \int \frac{e^x - e^{-x}}{t} \cdot \frac{dt}{e^x - e^{-x}}$$

$$= \int \frac{1}{t} \cdot dt$$

$$= \log|t| + C$$

$$= \log|e^x + e^{-x}| + C \quad (\text{Using } t = e^x + e^{-x})$$

$$= \boldsymbol{\log(e^x + e^{-x}) + C} \quad (\text{As } e^x + e^{-x} > 0)$$

Ex 7.2, 32

Integrate $\dfrac{1}{1+\cot x}$

Simplify the given function

$$\int \frac{1}{1+\cot x}\, dx$$

$$= \int \frac{1}{1+\frac{\cos x}{\sin x}}\, dx$$

$$= \int \frac{1}{\frac{\sin x+\cos x}{\sin x}}\, dx$$

$$= \int \frac{\sin x}{\sin x+\cos x}\, dx$$

Multiplying & dividing by 2

$$= \int \frac{2\sin x}{2(\sin x+\cos x)}\, dx$$

Adding & subtracting cos x in numerator

$$= \int \frac{\sin x + \sin x + \cos x - \cos x}{2(\sin x+\cos x)}\, dx$$

$$= \frac{1}{2}\int \left(\frac{\sin x+\cos x + \sin x - \cos x}{\sin x+\cos x}\right) dx$$

$$= \frac{1}{2}\int \left(\frac{\sin x+\cos x}{\sin x+\cos x} + \frac{\sin x-\cos x}{\sin x+\cos x}\right) dx$$

$$= \frac{1}{2}\int \left(1 + \frac{\sin x-\cos x}{\sin x+\cos x}\right) dx$$

$$= \frac{1}{2}\Bigg[x + \underbrace{\int \left(\frac{\sin x-\cos x}{\sin x+\cos x}\right) dx}_{\mathbf{I_1}}\Bigg] + C_1 \qquad ...(1)$$

Solving I_1

$$I_1 = \int \frac{\sin x - \cos x}{\sin x + \cos x}\, dx \qquad ...(2)$$

Let $\sin x + \cos x = t$

Differentiating both sides $w.r.t.x$

$$\cos x - \sin x = \frac{dt}{dx}$$

$$dx = \frac{dt}{\cos x - \sin x}$$

$$dx = \frac{dt}{-(\sin x - \cos x)}$$

$$I_1 = \int \frac{\sin x - \cos x}{\sin x + \cos x}\, dx = \int \frac{\sin x - \cos x}{t} \cdot \frac{dt}{-(\sin x - \cos x)}$$

$$= -1 \int \frac{dt}{t}$$

$$= -\log|t| + C$$

Putting back $t = \sin x + \cos x$

$$= -\log|\sin x + \cos x| + C_2$$

Putting the value of I_1 in (1)

$$\therefore \int \frac{1}{1 + \cot x} = \frac{1}{2}\left[x + \int \left(\frac{\sin x - \cos x}{\sin x + \cos x}\right) dx\right] + C_1$$

$$= \frac{1}{2}[x - \log|\sin x + \cos x| + C_2] + C_1$$

$$= \frac{x}{2} - \frac{1}{2}\log|\sin x + \cos x| + C_1 + \frac{C_2}{2}$$

$$= \frac{x}{2} - \frac{1}{2}\boldsymbol{\log|\sin x + \cos x| + C}$$

Ex 7.2, 34

Integrate $\frac{\sqrt{\tan x}}{\sin x \cos x}$

Concept:

There are two methods to deal with $\tan x$

(1) Convert into $\sin x$ and $\cos x$, then solve using the properties of $\sin x$ and $\cos x$.

(2) Change into $\sec^2 x$, as derivative of $\tan x$ is $\sec^2$.

Here, 1st Method is not applicable, so we have used 2nd Method.

Simplifying the function

$$= \frac{\sqrt{\tan x}}{\sin x \cos x . \frac{\cos x}{\cos x}}$$

$$= \frac{\sqrt{\tan x}}{\sin x . \frac{\cos^2 x}{\cos x}}$$

$$= \frac{\sqrt{\tan x}}{\cos^2 x . \tan x}$$

$$= (\tan x)^{\frac{-1}{2}} \times \sec^2 x$$

Integrating the function

$$\int \frac{\sqrt{\tan x}}{\sin x \cos x} . dx = \int (\tan x)^{\frac{-1}{2}} \times \sec^2 x . dx$$

Let $\tan x = t$

Differentiating both sides $w.r.t.x$

$$\sec^2 x = \frac{dt}{dx}$$

$$dx = \frac{dt}{\sec^2 x}$$

$$= \int (t)^{\frac{-1}{2}} . \sec^2 x . \frac{dt}{\sec^2 x}$$

$$= \int t^{\frac{-1}{2}} . dt$$

$$= \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C = 2t^{\frac{1}{2}} + C$$

$$= 2\sqrt{\tan x} + C$$

(Using $t = \tan x$)

Ex 7.2, 39

$\int \frac{dx}{\sin^2 x \cos^2 x}$ equals

(A) $\tan x + \cot x + C$

(B) $\tan x - \cot x + C$

(C) $\tan x \cot x + C$

(D) $\tan x - \cot 2x + C$

$$\int \frac{dx}{\sin^2 x \cos^2 x}$$

$$= \int \frac{1}{\sin^2 x \cos^2 x} . dx$$

$$= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} . dx \quad (Using \sin^2 x + \cos^2 x = 1)$$

$$= \int \frac{\sin^2 x}{\sin^2 x \cos^2 x} . dx + \int \frac{\cos^2 x}{\sin^2 x \cos^2 x} . dx$$

$$= \int \frac{1}{\cos^2 x} . dx + \int \frac{1}{\sin^2 x} . dx$$

$$= \int \sec^2 x . dx + \int cosec^2 x . dx$$

$$= \tan x - \cot x + C$$

Using $\int \sec^2 x . dx = \tan x$

and $\int cosec^2 x . dx = -\cot x$

∴ Option **B is correct.**

HOME ASSIGNMENT

EXERCISE-7.2

Q.NO-

8,10,20,23,27,33,35,36